Robotics and Autonomous Systems 57 (2009) 339–344

Contents lists available at ScienceDirect

Robotics and Autonomous Systems

journal homepage: www.elsevier.com/locate/robot

A Bayesian approach to information fusion for evaluating the measurement uncertainty

Klaus-Dieter Sommer [a,*], Olaf Kühn [b], Fernando Puente León [c], Bernd R.L. Siebert [a]

[a] Physikalisch-Technische Bundesanstalt (PTB), D-38116 Braunschweig, Germany

[b] Thuringian State Bureau for Metrology and Verification (LMET), D-98693 Ilmenau, Germany

[c] Universität Karlsruhe, Institut für Industrielle Informationstechnik, D-76128 Karlsruhe, Germany

ARTICLE INFO

Article history:
Available online 7 November 2008

Keywords:
Measurement uncertainty
Fusion
Bayes
Monte-Carlo
Probability density

ABSTRACT

The Bayesian approach to uncertainty evaluation is a classical example of the fusion of information from different sources. Basically, it is founded on both the knowledge about the measurement process and the influencing quantities and parameters. The knowledge about the measurement process is primarily represented by the so-called model equation, which forms the basic relationship for the fusion of all involved quantities. The knowledge about the influencing quantities and parameters is expressed by their degree of belief, i.e. appropriate probability density functions that usually are obtained by utilizing the principle of maximum information entropy and the Bayes theorem. Practically, the Bayesian approach to uncertainty evaluation is put into effect by employing numerical integration techniques, preferably Monte-Carlo methods. Compared to the ISO-GUM procedure, the Bayesian approach does not have any restrictions with respect to nonlinearities and calculation of confidence intervals.

1. Introduction

The concept which, in accordance with the "Guide to the Expression of Uncertainty in Measurement" [1] (here denoted as ISO-GUM), underlies the modern evaluation of measurement uncertainty is based on both the available information about the measuring process and the (input) quantities and parameters that have an influence on the value of the measurement result. The knowledge about the measuring process is to be condensed to the so-called model equation (fusion model). It mathematically represents the interrelation between the relevant measurand Y, the involved input quantities $X_1, \ldots, X_N$ as well as their values, ξ_i for X_i and η for Y, respectively:

$$Y = f_{\mathrm{M}}(X_1, \ldots, X_N), \tag{1}$$

$$\eta = f_{\mathrm{M}}(\xi_1, \ldots, \xi_N). \tag{2}$$

The ISO-GUM [1] mainly considers only one output quantity (measurand) Y. However, the concept can easily be extended to more than one output quantity; a simple example can be found in Annex H2 of the ISO-GUM [1], and a general procedure in the standard DIN 1319-4 [2].

The output quantity Y is the measurand, and the input quantities X_i are quantities or parameters that can influence the result that is obtained for the measurand.

Basically, it is the aim of measurement data and uncertainty evaluation to determine both the best estimate of the measurand and an associated measurement uncertainty. Therefore, the so-called *model equation* used in uncertainty evaluation is to be clearly distinguished from the so-called *measurement equation* that represents the cause-effect relationship and is often used in sensor techniques. Hence, measurement data and uncertainty evaluation usually raise a so-called inverse problem. Fig. 1 illustrates this and the resulting different relationships represented by these two model categories [3].

The model equation might be understood as a fusion model for the information about the relevant input quantities $X_1, \ldots, X_N$.

Moreover, because the model equation is presumed to represent the relationship between the input quantities and the measurand uniquely and completely, it additionally covers the scaling of the input quantities.

Any information about these input quantities is to be weighed as more or less relevant and reliable by assigning appropriate state-of-knowledge probability density functions (pdfs) $g_{X_i}(\xi_i)$ to them, where ξ_i are the possible values of the quantities X_i. In accordance with the Bayesian concept [4,5], a pdf represents the degree of belief about the individual input quantity from wherever this knowledge is descended.

In contemporary uncertainty evaluation, the expectation value of a pdf, $x_i = \mathrm{E}[X_i]$, is taken for the best estimate of the quantity, and the standard deviation of the pdf is taken as the standard uncertainty associated with the above expectation, $u_{x_i} = \sqrt{\mathrm{Var}[X_i]}$.

* Corresponding author. Tel.: +49 531 592 3010; fax: +49 531 592 3015.
E-mail address: klaus-dieter.sommer@ptb.de (K.-D. Sommer).

0921-8890/$ – see front matter © 2009 Published by Elsevier B.V.
doi:10.1016/j.robot.2008.10.011

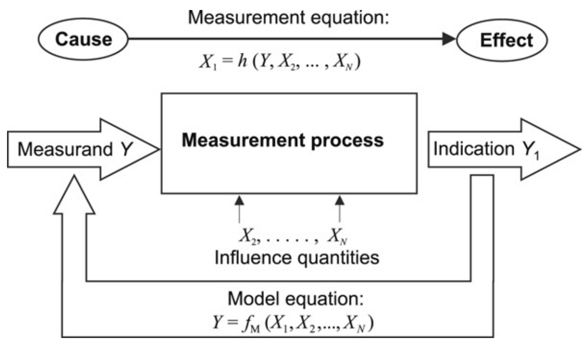

Fig. 1. Model categories: Cause-effect relationship ("measurement equation") and "model equation used for the uncertainty evaluation".

2. Bayesian description of knowledge about input quantities

The state-of-knowledge pdf for any input quantity X_j may be obtained by utilizing the principle of maximum information entropy (pme) [4] that, for example, yields

- A rectangular pdf if one knows that the values ξ_i of the quantity X_i are contained in an interval (practical examples: given tolerances or error limits, digital resolution),
- A Gaussian (normal) pdf if one knows the best estimate $x_i = \mathrm{E}[X_j]$ and the associated standard uncertainty u_{x_j} of the quantity X_j (practical examples: statement of a calibration result, result of a statistical analysis expressed by a mean and a standard deviation).

If new or additional information I_2 is available, the change of a (possibly) given prior pdf is described by the Bayes theorem [4–8]: the so-called posterior pdf $g(\xi|I_2, I_1)$ taking account of new data I_2 results from the prior pdf $g(\xi|I_1)$ based on prior information I_1 as a product of a normative constant C, the likelihood $l(\xi|I_1, I_2)$, and the prior pdf as follows:

$$g(\xi|I_2, I_1)\mathrm{d}\xi = C \cdot l(\xi|I_1, I_2) \cdot g(\xi|I_1)\mathrm{d}\xi. \tag{3}$$

For the case of existing prior information about the measurand and a repeatedly observed indicated quantity Q which (presumably completely) reflects the measurand Y, the Bayesian approach to measurement data and uncertainty evaluation is illustrated, as an example, in Fig. 2. For the sake of simplification, this example does not take into account any additional knowledge about systematic effects; hence the likelihood is equal to the frequency distribution of the observed data.

It should be clearly noted that this method of uncertainty evaluation is not (yet) part of the ISO-GUM procedure [1], but the Joint Committee for Guides in Metrology (JCGM) of *Bureau International de Poids et Mesures* (BIPM) is preparing further documents that are based consistently on Bayesian probability theory [9]. It recently has released a Supplement 1 to the ISO-GUM: "Propagation of distributions using a Monte-Carlo method" [10].

The influence of prior information on the resulting measurement uncertainty is demonstrated quantitatively with a simple example that is depicted in Fig. 3. For this example, it is assumed that the standard uncertainty associated with the estimated value of the measurand (prior knowledge) is more or less of the same order as the standard deviation (of the mean) of repeatedly observed data; i.e. the standard deviation of the frequency distribution that is equal to the likelihood.

Consequently, Fig. 3 shows that, with an increasing number of observations, which results in a decreased standard deviation of the mean of the observed data, the likelihood significantly gains influence and, therefore, the influence of the prior loses ground.

From Eq. (17) in Section 4, the following relationship of the involved standard uncertainties and standard deviations, respectively, can be derived:

$$\frac{1}{u_{\mathrm{post}}^2} = \frac{1}{u_{\mathrm{prior}}^2} + \frac{1}{s_{\mathrm{hQ}}^2}, \tag{4}$$

where u_{prior} is the standard uncertainty associated with the prior estimate of the value of the measurand, s_{hQ} is the standard deviation (of the mean) of the frequency distribution of the observed data, and u_{post} is the resulting standard uncertainty.

For the posterior uncertainty, this yields

$$u_{\mathrm{post}} = \sqrt{\frac{u_{\mathrm{prior}}^2 \cdot s_{\mathrm{hQ}}^2}{u_{\mathrm{prior}}^2 + s_{\mathrm{hQ}}^2}}. \tag{5}$$

In contrast to the ISO-GUM approach [1], the Bayesian approach leads to an uncertainty weighted combination of the expectation values of the involved quantities. One obtains

$$\frac{y_{\mathrm{post}}}{u_{\mathrm{post}}^2} = \frac{y_{\mathrm{prior}}}{u_{\mathrm{prior}}^2} + \frac{x_{\mathrm{likelih}}}{s_{\mathrm{hQ}}^2}, \tag{6}$$

and hence

$$y_{\mathrm{post}} = u_{\mathrm{post}}^2 \cdot \left[\frac{y_{\mathrm{prior}}}{u_{\mathrm{prior}}^2} + \frac{x_{\mathrm{likelih}}}{s_{\mathrm{hQ}}^2}\right]. \tag{7}$$

3. Repeated observations

Today, in practice, the prior knowledge about the measurand itself is usually neglected. Therefore, this common case is described here in more detail. In the case of repeated observations $q_1, \ldots, q_n$ of the quantity Q, a Gaussian probability model for any given datum q_k yields [5,6,11]:

$$g(q_k|\varpi, \sigma_Q) \propto \sigma_Q^{-1} \cdot \exp\left[-\frac{(q_k - \varpi)^2}{2\sigma_Q}\right], \tag{8}$$

where ϖ represents the possible values of Q, and σ_Q are the possible values of the standard deviation associated with Q. In absentia of systematic effects, the above pdf may be assumed to be equal to the frequency distribution for the observed data and is usually interpreted as being proportional to the likelihood function $l(\varpi, \sigma_Q|Q)$ [5,6,11], that is

$$l(\varpi, \sigma_Q|Q) \propto \sigma_Q^{-n} \cdot \exp\left[-\frac{\chi^2(\varpi, \sigma_Q, Q)}{2}\right], \tag{9}$$

where

$$\chi^2 = \sum_{k=1}^{n}\left(\frac{q_k - \varpi}{\sigma_Q}\right)^2 = \frac{n}{\sigma_Q^2}\left[(\varpi - \bar{q})^2 + \frac{n-1}{n}s_q^2\right],$$

$$s_q = (n-1)^{-1} \cdot \sum_{k=1}^{n}(q_k - \bar{q})^2$$

and

$$\bar{q} = n^{-1} \cdot \sum_{k=1}^{n} q_k,$$

where s_q is the standard deviation of the observed data and $\bar{q}$ is the mean.

By multiplying Eq. (9) with the non-informative Jeffrey's prior, the joint posterior pdf is obtained [5,6,11]:

$$g(\varpi, \sigma_Q|Q) \propto \sigma_Q^{-(n+1)} \cdot \exp\left[-\frac{\chi^2(\varpi, \sigma_Q, Q)}{2}\right]. \tag{10}$$

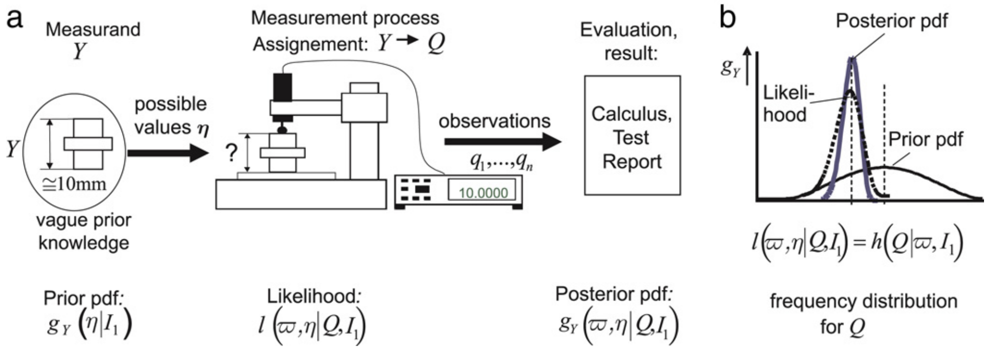

Fig. 2. Simplified example of Bayesian inference in measurement: (a) Generalized measurement process. (b) Probabilistic description of the state of knowledge. Symbols: $g_Y(\eta|I_1)$ – prior pdf representing vague prior knowledge I_1 about the measurand, e.g. the nominal value and given error limits for the measurand; $l(\varpi,\eta|Q,I_1)$ – likelihood representing the measuring process with the observed quantity Q; $g_Y^*(\varpi,\eta|Q,I_1)$ – posterior pdf representing the available knowledge inferred from the likelihood and the prior pdf; $h(Q|\varpi,I_1)$ – frequency distribution for the quantity Q inferred from the observed values $q_1,\ldots,q_n$; ϖ – possible values of the quantity Q; η – possible values of the measurand Y.

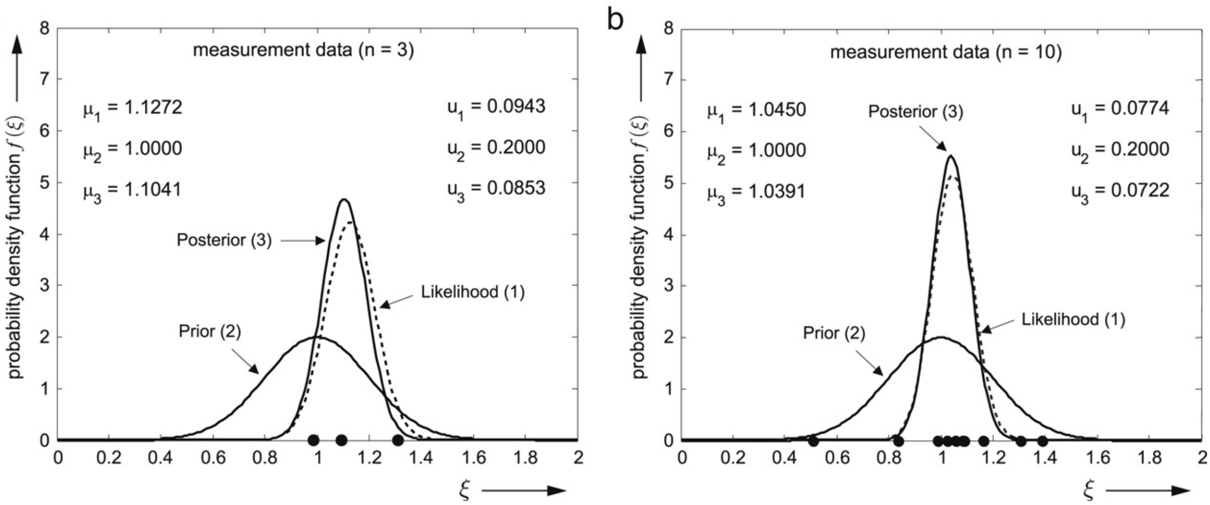

Fig. 3. Improvement of posterior pdf (3) calculated with Eq. (5) and (7) from the likelihood of the observed measurement data (big black points on x-axis) given a prior distribution (2); μ_1, μ_2, μ_3 and u_1, u_2, u_3 are parameters of the Gaussian distribution corresponding to the pdf (1), (2), (3); (a) number of a measurement data ($n = 3$); (b) number of a measurement data ($n = 10$).

Integration to dσ leads to the information pertaining to the expectation for Q:

$$g(\varpi|Q) \propto \left\{1+\frac{[(\varpi-\bar{q})/s(\bar{q})]^2}{n-1}\right\}^{-n/2}, \tag{11}$$

where $s(\bar{q}) = s(q_k)\cdot n^{-1/2}$.

Since the right-hand side of Eq. (11) obviously corresponds to a Student-t distribution, the t-distributed variable, i.e. the ratio of the sum of the individual sample deviations and the sample standard deviation, $T = (Q-\bar{q})\cdot s^{-1}(\bar{q})$, is introduced.

One obtains

$$g(t) \propto \left(1+\frac{t^2}{n-1}\right)^{-n/2},$$

where t are the possible values of T. Therefore, the best estimate for Q, i.e. the estimate of the measurand, becomes

$$q = \mathrm{E}[Q] = \bar{q} = \frac{1}{n}\sum_{k=1}^{n} q_k. \tag{12}$$

Due to $\mathrm{Var}[T] = (n-1)(n-3)^{-1}$, the "Bayesian uncertainty contribution" associated with the expectation of the repeatedly observed quantity Q becomes [5,11]

$$u_Q = \sqrt{\frac{n-1}{n(n-3)}}\cdot s(q_k). \tag{13}$$

It should be mentioned that, for small numbers of observations ($n \le 12$), this uncertainty contribution significantly exceeds the so-called type-A uncertainty, calculated in accordance with the ISO-GUM [1]. The GUM-type-A uncertainty, therefore, may be understood as an approximation for a sufficiently large number of observations [12].

4. Joint posterior distribution for the measurand

It is the intrinsic purpose of the Bayesian approach to uncertainty analysis to develop the joint posterior pdf for the output quantity (measurand) which is compatible with the given information about the (values of the) input quantities and the measuring process:

$$g_Y(\eta|Q,I) \propto l(\boldsymbol{\xi}|Q,I)\cdot g(\boldsymbol{\xi}|I), \tag{14}$$

where $g(\boldsymbol{\xi})$ represents the state of knowledge about the values of the input quantities $\boldsymbol{\xi} = \xi_1,\ldots,\xi_N$.

Because of the interrelation of the input quantities (given by the Eq. (1) and (2)), *a posteriori* the input quantities cannot be acknowledged as being independent. This fact can be taken into consideration by writing the *joint prior* pdf as [6,11]

$$g(\boldsymbol{\xi}|I) \propto g_X(\boldsymbol{\xi})\cdot g_{\mathrm{M}}(\boldsymbol{\xi}), \tag{15}$$

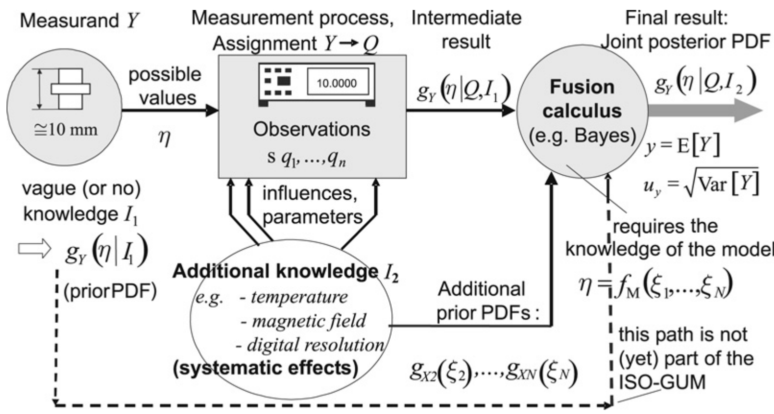

Fig. 4. Generalized measurement process as Bayesian inference by means of model-based fusion of the state-of-knowledge pdfs for the input quantities. Symbols: see text (depicted according to Beyerer [14]).

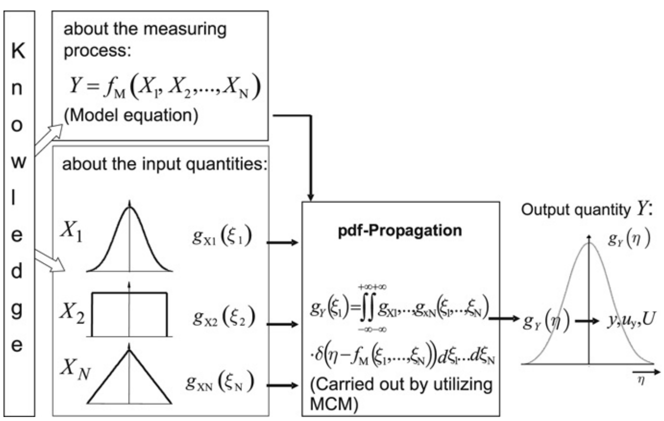

Fig. 5. Bayesian approach to uncertainty evaluation by propagating the pdfs for the input quantities [9,10] as a step-by-step procedure.

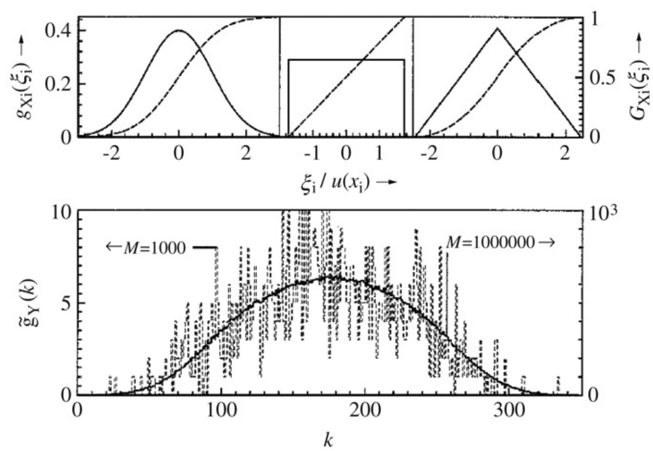

Fig. 6. Illustration of computing a (frequency) distribution for the output quantity by means of Monte-Carlo techniques [17]. The upper graphs show a Gaussian, a rectangular and a triangular PDF and (right ordinate) the corresponding distribution functions. Representative draws $\xi_{G,r}$, $\xi_{T,r}$ and $\xi_{T,r}$ are made and the corresponding model function formed. Doing so yields a representative draw, $\eta_r = f(\xi_{G,r}, \xi_{R,r}, \xi_{T,r})$, from the PDF for Y. The complete set of such draws is sorted to yield a histogram representation of the frequency distribution. The bottom graph shows the resulting frequency distribution obtained for $M = 10^3$ and (right ordinate) $M = 10^8$ draws [9,10,17].

where $g_X(\boldsymbol{\xi})$ is the prior of the input quantities and $g_M(\boldsymbol{\xi})$ the so-called model prior [11].

Modelling is admittedly the hardest part in assessing uncertainty, since a theory on modelling does not exist. But an appropriate model equation may be obtained by systematically analyzing the cause-and-effect chain of the measurement, including all relevant influences and disturbances [3,13].

For models of the form $M(X) = Y - f_M(X) = 0$, the above mentioned model prior is equal to Dirac's delta function [11]:

$$g_M(\boldsymbol{\xi}) = \delta[M(\eta)], \tag{16}$$

that "takes care" that only meaningful combinations of the possible values of the input quantities are taken into consideration ("filter function"). Therefore the joint posterior pdf for the output quantity becomes

$$g_Y(\eta|Q, I) = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} g_{X_1}, \dots, g_{X_N}(\xi_1, \dots, \xi_N) \times \delta(\eta - f_M(\xi_1, \dots, \xi_N)) \mathrm{d}\xi_1, \dots, \mathrm{d}\xi_N. \tag{17}$$

Eq. (17) is known as *Markov formula*. Fig. 4 illustrates the (fully) Bayesian concept for evaluating the measurement result and its associated uncertainty by means of a generalized measurement process, and Fig. 5 shows this Bayesian approach to the calculation of measurement uncertainty as a straightforward step-by-step procedure.

Since the Markov formula can be analytically computed in fairly simple cases only, modern uncertainty evaluation utilizes Monte-Carlo techniques (MCM) as integration techniques for pdf propagation [15–17]. MCM appears to be the "natural way" for combining uncertainties. It is therefore recommended by the BIPM as a general method for evaluating uncertainty [9,10]. Fig. 6 illustrates the Monte-Carlo method as usually applied in metrology.

5. Expectation, uncertainty and expanded uncertainty for the output quantity

From the pdf for the output quantity $g_Y(\eta|Q, I)$, the expectation value of the measurand $y = \mathrm{E}[Y]$ and its associated uncertainty u_y can be derived:

$$y = \int_{-\infty}^{\infty} g_Y(\eta)\eta \mathrm{d}\eta, \tag{18}$$

and

$$u_y = \sqrt{\int_{-\infty}^{\infty} g_Y(\eta)(\eta - y)^2 \mathrm{d}\eta}. \tag{19}$$

Since the propagation of distributions [9,10] does explicitly provide the pdf for the output quantity, the expanded uncertainty, i.e. a kind of confidence interval, can easily be derived as the

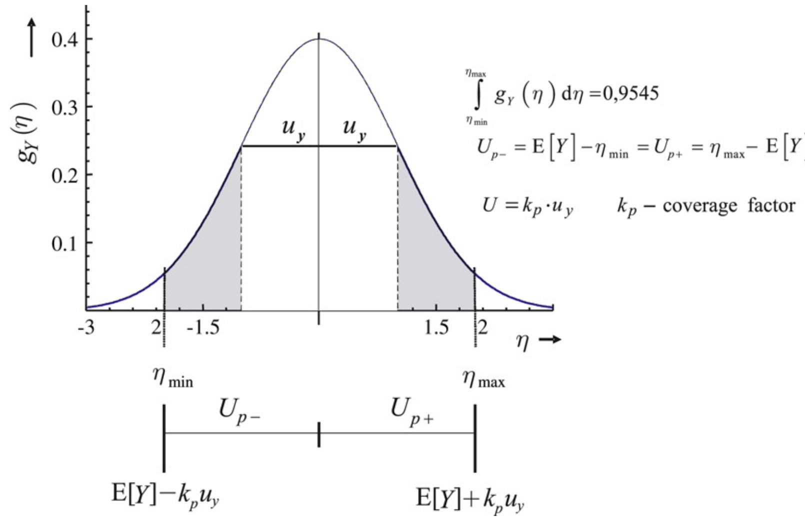

Fig. 7. Illustration of the coverage interval for a given output quantity PDF.

minimum interval $[U_{P-}; U_{P+}]$ that meets the following coverage probability condition (see Fig. 7):

$$\int_{-\infty}^{U_{P+}} g_Y(\eta)\mathrm{d}\eta - \int_{-\infty}^{U_{P-}} g_Y(\eta)\mathrm{d}\eta = P. \tag{20}$$

In metrology, usually, this coverage probability P is set up to minimum 0.95 [1].

6. Linear models (linear model equations)

In practice, users of a measurement result will often not be interested in the pdf for the output quantity Y but rather in its expectation value y and the associated measurement uncertainty u_y (see Eq. (18) and (19)).

In the case of linear or linearized model equations, e.g. by first-order Taylor series expansion, the Markov formula directly results in the Gaussian rule of uncertainty propagation:

$$y = f_{\mathrm{M}}(x_1, \ldots, x_N), \tag{21}$$

$$u_y = \sqrt{\sum_{i=1}^{N} \left(\left. \frac{\partial f_{\mathrm{M}}}{\partial X_i} \right|_{x_i} \right)^2 u_{x_i}^2 + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left. \frac{\partial f_{\mathrm{M}}}{\partial X_i} \right|_{x_i} \left. \frac{\partial f_{\mathrm{M}}}{\partial X_j} \right|_{x_j} u_{x_i x_j}}, \tag{22}$$

where $y = \mathrm{E}[Y]$; $u_{x_i x_j} = u_{x_i} \cdot u_{x_j} \cdot r(X_i; X_j)$ is the estimated covariance of the quantities X_i and X_j, and $r(X_i; X_j)$ is the respective correlation coefficient.

It is a common experience that, in the majority of practical uncertainty evaluations, the ISO-GUM procedure will provide satisfying results [12,18]. But besides nonlinearity, the calculation of the expanded measurement uncertainty is a real weak point of the standard concept [12,18]. The problem is caused by the fact that the standard procedure does not provide the pdf for the output quantity and, therefore, the coverage factor needed to calculate the expanded uncertainty is to be determined on the basis of only vague information about this pdf:

$$k_{\mathrm{P}} = U \cdot u_y. \tag{23}$$

7. Conclusion

It becomes clear that, independent on the calculus used (Gaussian or Bayesian), for practitioners, the key steps of modern uncertainty evaluation are the compilation and description of the knowledge regarding the measurement, the modelling of the measurement, and the assignation of an appropriate pdf to each of the involved input quantities.

It can be concluded that the Bayesian approach allows for stringently evaluating the measurement uncertainty. There are no restrictions related to nonlinearity and determination of the expanded uncertainty. On the other hand, with the exceptions of evaluating the expanded uncertainty and calculating the standard uncertainty from only a few repeated observations, the ISO-GUM procedure is (for linearizable systems) consistent with the Bayesian concept [12,18].

References

[1] Guide to the Expression of Uncertainty in Measurement (GUM), First edition, 1993, corrected and reprinted 1995, International Organization for Standardization (ISO), Geneva, 1993 and 1995.
[2] DIN 1319-4:1999-02. Grundlagen der Messtechnik – Teil 4: Auswertung von Messungen; Messunsicherheit, Beuth Verlag.
[3] K.-D. Sommer, B.R.L. Siebert, Systematic approach the modelling of measurements for uncertainty evaluation, Metrologia 43 (2006) 4.
[4] K. Weise, W. Woeger, A Bayesian theory of measurement uncertainty, Measurement Science & Technology 3 (1992) 1–11.
[5] K. Weise, W. Woeger, Messunsicherheit und Messdatenauswertung in der Metrologie, WILEY-VCH, Weinheim, 1999.
[6] I. Lira, W. Woeger, Bayesian evaluation of the standard uncertainty and coverage probability in a simple measurement model, Measurement Science Technology 12 (2001) 1172–1179.
[7] D.S. Sivia, Data Analysis — A Bayesian Tutorial, Clarendon Press, Oxford, 1996.
[8] T.W. Estler, Measurement as inference: Fundamental ideas, Annals of the CIRP 48 (2) (1999) 611–632.
[9] W. Bich, M.G. Cox, P.M. Harris, Evaluation of the Guide to the Expression of Uncertainty in Measurement, Metrologia 43 (4) (2006).
[10] Evaluation of measurement data – Supplement 1 to the Guide to the expression of uncertainty in measurement – Propagation of distributions using a Monte Carlo method, www.bipm.org/en/publications/guides/gum.html.
[11] I. Lira, Evaluating the Measurement Uncertainty, Institute of Physics Publishing Bristol and Philadelphia Ltd., 2002.
[12] R. Kacker, A. Jones, On use of Bayesian statistics to make the Guide to the Expression of Uncertainty in Measurement consistent, Metrologia 40 (2003) 235–248.
[13] K.-D. Sommer, B.R.L. Siebert, M. Kochsiek, A. Weckenmann, A systematic approach to the modelling of measurements for uncertainty evaluation, in: 7th International Symposium on Measurement Technology and Intelligent Instruments ISMTII 2005, Journal of Physics: Conference Series 13 (2005) 224–227.
[14] J. Beyerer, The value of additional knowledge in measurement — a Bayesian approach, Measurement 25 (1999) 1–7.
[15] M.G. Cox, B.R.L. Siebert, The use of Monte-Carlo method for evaluating uncertainty and expanded uncertainty, Metrologia 43 (2006) 4.
[16] B.R.L. Siebert, P. Ciarlini, D. Sibold, Monte Carlo Study on Logical and Statistical Correlation in Advanced Mathematical & Computational Tools in Metrology V, in: P. Ciarlini, E. Filipe, A.B. Forbes, F. Pavese, C. Perruchet, B.R.L. Siebert (Eds.), Series on Advances in Mathematics for applied Sciences, vol. 57, World Scientific, New Jersey, 2006, pp. 235–244.
[17] B.R.L. Siebert, K.-D. Sommer, Weiterentwicklung des GUM und Monte-Carlo-Techniken/new developements of the GUM and Monte Carlo techniques, Technisches Messen 71 (2) (2004) 67–80.
[18] R. Kacker, K.-D. Sommer, R. Kessel, Evolution of modern approaches to express uncertainty in measurement, Metrologia 44 (2007) 513–529.

Klaus-Dieter Sommer received his degree as Diplomated Engineer (Dipl.-Ing) in electrical engineering and automatic control from the Otto-von-Guericke University of Magdeburg in 1974. In 1984 he received his Doctorate in electrical engineering and measurement techniques from Ilmenau Technical University. From 1983 to 1993 he was in charge of density metrology in both the former ASMW and, later, the Physikalisch-Technische Bundesanstalt (PTB). From 1994 to 2007, he was the Director of the Thuringian State Bureau for Metrology and Verification in Ilmenau. Since 2007 he has been the head of PTB division 3 on Chemical Physics and Explosion Protection. Additionally, Klaus-Dieter Sommer is Honorary Professor for Measurement Techniques of the University of Erlangen-Nuremberg and visiting lecturer for Measurement Data Evaluation of the Braunschweig Technical University. His research interests include modelling of measurements for uncertainty analysis, uncertainty evaluation in chemical analysis and measurement theory.

Olaf Kühn studied Information Technology at the Technische Universität Ilmenau. He received his Ph.D. in 1997 with a thesis on non-contact optical precision measurement. He was scientific assistant at the Department of Quality Assurance of the Faculty of Mechanical Engineering at the Technische Universität Ilmenau from 1997 to 2004. Since 2004 he has been the head of the Ilmenau Metrological Test Office at the Thuringian State Bureau of Metrology and Verification (LMET). He is a quality manager at the German Association of Quality Management. Since 2005 he has been a visiting lecturer for Image Processing at the Technische Universität Ilmenau. Since 2007 he became the head of Thuringian State Bureau of Metrology and Verification (LMET). His research interests focus on metrology, image processing, robust feature extraction and data fusion.

Fernando Puente León received the M.S. degree in electrical engineering and the Ph.D. degree in automated visual inspection from the University of Karlsruhe, Karlsruhe, Germany, in 1994 and 1999, respectively. He heads the Institute of Industrial Information Technology at the University of Karlsruhe as an Ordinary Professor. From 2003 to 2008 he was a Professor with the Department of Electrical Engineering and Information Technology, Technische Universität München, München, Germany. From 2001 to 2002, he was with the Design of Systems on Silicon, Valencia, Spain. From 2002 to 2003, he was a Postdoctoral Research Associate with the Institut für Mess- und Regelungstechnik, University of Karlsruhe, where he was the Head of the Information Fusion Group. His research interests include image processing, automated visual inspection, information fusion, measurement technology, pattern recognition, and communications.

Bernd R.L. Siebert received the Ph.D. degree in nuclear theory from the Rensselaer Polytechnic Institute (RPI) in Troy (NY, USA) in 1973 and spent his professional life until his retirement in 2006 at the Physikalisch-Technische Bundesanstalt (PTB) in Braunschweig (Germany). From 1984 until 1997 he was the Head of the Laboratory Theoretical Neutron Metrology. After the closure of the PTB research reactor he became the Head of the Laboratory Numerical Physics. His research interest focused first on numerical methods for neutron dosimetry, especially for designing virtual instruments and simulating complex measurements for evaluation and optimisation. In recent years he developed Monte Carlo Techniques for the evaluation uncertainty, with an emphasis on multivariate problems.